Understanding the Design Space for Cognitive Networks

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Abstract—This paper studies a cognitive network where licensed primary users and unlicensed but 'cognitive' secondary users share spectrum. Many system design parameters affect the joint performance, e.g., outage and capacity, seen by the two user types in such a scenario. We explore the sometimes subtle system tradeoffs that arise in such networks. To that end, we propose a new simple stochastic geometric model that captures the salient interdependencies amongst spatially distributed primary and secondary nodes. The model allows us to evaluate the performance dependencies between primary and secondary transmissions in terms of the outage probability, node density and transmission capacity. From the design perspective the key design parameters determining the joint transmission capacity and tradeoffs, are the detection radius (detection SINR threshold), decoding SINR threshold, burstiness of coverage and/or transmit powers. We show how the joint transmission capacity region can be optimized or affected by these parameters.

Index Terms—cognitive network, stochastic geometry, network information theory, transmission capacity

I. INTRODUCTION

FCC and researchers have observed the scarcity and the underutilization of spectrum resources which suggests that a new model of spectrum usage is required, usually referred to as cognitive radio/network, see e.g., [1]. The basic approach is to allow unlicensed or secondary devices to opportunistically access a spectrum allocated to licensed or primary devices. The focus of this paper is on the transmission capacity of cognitive networks, in particular on characterizing the spatial or temporal spectrum opportunities for secondary devices and their interaction with primary devices.

Specifically, we model primary transmitters (PTx) corresponding to high-power broadcasting towers, e.g., a fixed or mobile TV broadcasting station, sending the same signal to multiple primary receivers (PRx). This is usually called a single frequency network (SFN). The coverage of a single PTx is relatively large, e.g., tens of kilometers, and receivers can successfully decode the signal if they belong to the coverage area of at least one transmit station. Signals from different stations are treated as delayed multi-path. Cognitive or secondary devices can transmit in regions where the primary signal is not detected. PTxs are not aware of the existence of secondary devices and the same secondary network characteristics are assumed where secondary transmitters (STx) and receivers (SRx) are involved in ad-hoc or peer-to-peer low power transmissions.

Related work: In [2] and numerous subsequent papers (see survey in [3]) various spatial models have been introduced where nodes are randomly distributed on a plane and signal attenuation is a function of an attenuation factor and of the distance between transmitter and receiver. In addition, [4], [5] and

[6] explored the capacity of networks in terms of transmission capacity, which measures transmitted bits per second per meter square. Their models capture the subtle interactions between nodes in terms of outage, so they allow the computation of the exact capacity rather than the scaling behavior. However, most of this work focuses on capacity analysis for *homogeneous* networks.

Recently, above methodologies have been extended to evaluate the performance of multiple networks with different priorities in the context of cognitive networks; for example, [7], [8] focus on scaling laws for "two networks with different access priority". In their work, primary and secondary networks are found to have the same capacity scaling law $\Theta(\sqrt{n/\log n})$ and $\Theta(\sqrt{m/\log m})$ where n and m correspond to the primary/ secondary receiver densities. [9] studied the impact of transmission power of secondary nodes on the reliability of detection performance and the transmission opportunity for secondary nodes. However, this work considers only a single secondary node with randomly distributed multiple primary nodes. Overlaid spectrum sharing between two different networks was studied in [10], where the mobile ad-hoc devices are overlaid with uplink transmissions of an existing cellular network and the capacity trade-off between two networks was characterized. However, in this work, the secondary nodes do not have spatial detection or cognitive function. [11] studied cognitive networks with single primary and multiple secondary transmitters. A bound on the radius of the primary exclusive region, i.e., where the primary transmitter can communicate with its receivers under an outage constraint, was found based on various system parameters. [12] considered a cognitive network where nodes access medium using CSMA protocol with two different types of access priorities.

Contributions: In this paper, we model *both* primary and secondary devices as point processes, which allows us to capture the impact from both PTxs and STxs to both PRxs and SRxs. We also model the cognitive operation of secondary devices; as a result, the two processes are *dependent* on each other. Our model delivers rich insights on system performance and design tradeoffs in terms of coverage, node density, outage probability, and capacity. Our contributions can be summarized as follows.

First, we provide a novel and mathematically tractable Boolean disk model for primary and secondary networks, which is simple yet captures the stochastic nature of the interaction between the two networks. The coverage reduction of PTxs and the impact of hidden PTxs on outage, node density and capacity of STxs are characterized.

Second, we identify several important design parameters: detection radius (or detection SINR threshold), decoding SINR threshold and transmit power of STx, which affect the achievable capacity of the secondary network. It is shown that a

conservatively selected detection radius can severely decrease the capacity of a secondary network and that the optimal decoding SINR of SRxs depends on the density of PTxs. We also show that a secondary network with a conservative detection radius can achieve higher capacity if the associated primary network has more bursty coverage. While an ideally chosen detection radius makes the achievable capacity of the secondary network be independent of the burstiness of primary network's coverage, it does introduce interference to PRxs. We provide *rules of thumb* on how to tune these design parameters to maximize the capacity of the cognitive network.

II. System Model

A. Preliminary Definitions

We first define the notation that will be used throughout this paper. $b(x, r)$ denote a ball centered at location $x \in \mathbf{R}^2$ with radius r. Let $\|x - y\|$ denote a distance between two points x and y in $\mathbf{R}^2$ and $|A|$ denote the area of set $A \subset \mathbf{R}^2$. For a random variable Q, let $\mathcal{L}_Q(s) \equiv \mathbb{E}\left[e^{-sQ}\right]$ denote the Laplace transform of the random variable Q.

B. Path loss and Interference Model

We assume free space path loss model $d^{-\alpha}$ given an attenuation factor α and distance d between transmitter/interferer and receiver. When SINR is computed, only the dominant interferer is considered. If the dominant interferer is within the interference radius of the receiver, the receiver fails to receive; otherwise, the interferer is ignored. The interference radius is conservatively determined based on various factors such as interference power, signal power, noise and the receiver's decoding SINR. In our interference model, we do not take into account the additive nature of interference. Indeed this so-called protocol interference model is widely used [2], [13] and this model produces asymptotically tight estimates [5], [6].

C. Primary Network Model

The primary network consists of a set of PTx-PRx pairs. We model only the locations of PTxs which for simplicity are assumed to follow a homogeneous Poisson point process (HPPP) $\Pi_p = \{X_i\}$ in $\mathbf{R}^2$ with intensity λ_p. We use X_i to denote both the i-th PTx and its location in $\mathbf{R}^2$. PTxs transmit with the same transmission power ρ_p and realize a rate $b_p = \log(1 + \beta_p)$ bps where β_p is the SINR threshold to decode PTx's signal. A PRx Y can decode the signal from PTxs if it is within PTxs' coverage area, $B(\Pi_p, d_p) \equiv \cup_{X_i \in \Pi_p} b(X_i, d_p)$ and, at the same time, is not interfered by a STx; here d_p denotes the target coverage radius of PTxs. A PRx Y can be interfered by STxs if one or more active STxs exist within its interference region $b(Y, r_{sp})$, where r_{sp} is STx's interference radius of a PRx w.r.t to a STx.

D. Secondary Network Model

The secondary network consists of a set of STx-SRx pairs. The STxs are modeled by HPPP $\Pi_s = \{Z_i\}$ with intensity λ_s[1]. We assume that all the STxs sense the medium at the same time and only STxs which detect the absence of PTxs attempt to transmit in Aloha fashion. This again represents a strong simplification. So, it is possible that a SRx is interfered by one or more other active STxs, causing an outage. Indeed, this model can be viewed as a snapshot of active secondary nodes at a typical time slot. Note that not all the STxs are allowed to transmit since some of them are blocked out by PTxs and accordingly inactive. We assume that a STx uses a simple *signal energy detection* scheme to detect whether there are PTxs within its detection radius r_d. A SRx W is interfered by PTxs if one or more PTx exist within $b(W, r_{ps})$, where r_{ps} is the interference radius of a SRx w.r.t a PTx. For a given primary process Π_p, we model the active STxs by a point process $\Pi_s^a = \Pi_s^a(\Pi_p) = \{Z_i \in \Pi_s | Z_i \notin B(\Pi_p, r_d)\}$ with intensity $\lambda_s^a(z, \Pi_p) = \lambda_s \mathbf{1}\{z \notin B(\Pi_p, r_d)\}$ at $z \in \mathbf{R}^2$. Note that Π_s^a is a stationary doubly stochastic or Cox process with a random intensity measure [15]. Also note that for a given π_p, a realization of Π_p, this process of active STx corresponds to a thinned process, where the thinning is spatially correlated depending on the π_p. Thus, the resulting process is a non-HPPP. We assume that a STx transmits to a SRx which is located a fixed distance d_s away with transmission power ρ_s. Like PTx, a STx transmits $b_s = \log(1 + \beta_s)$ bps, where β_s is the SINR threshold to decode STx's signal. The STx's signal can interfere with both PRxs and un-intended SRxs; that is, STx Z_i in $b(Y, r_{sp})$ can interfere with PRx Y and STx Z in $b(W, r_{ss})$ can interfere with SRx W, where r_{sp} and r_{ss} are the interference radii of a PRx and a SRx w.r.t. a STx respectively.

E. System Model Parameters

In this section, we discuss the system parameter selection. We shall assume that β_p, β_s, and the tolerable interference I_p are specified as part of the system design. I_p corresponds to the amount of interference that PRxs can tolerate at the edge of PTxs' coverage and can be understood as a performance margin to overcome uncertainty in noise and interference. Given these parameters, the following system parameters can be determined. We first determine PTx's maximum coverage radius d_p from PRx's successful reception condition, i.e, if a PRx receives successfully, then its received SINR, assuming noise η and maximum tolerable interference I_p at the coverage edge, should be larger than the decoding SINR threshold, which gives following:

$$d_p \equiv \sup\left\{d > 0 | \frac{\rho_p d^{-\alpha}}{\eta + I_p} > \beta_p\right\} = \left(\frac{\rho_p}{(\eta + I_p)\beta_p}\right)^{\frac{1}{\alpha}}.$$

Second, if PRx receives PTx's signal successfully, then, the SINR at the above receiver should be larger than β_p even when interference from STx is considered: $\frac{\rho_p d^{-\alpha}}{\eta + \rho_s r^{-\alpha}} > \beta_p$, this allows us to define a PRx's interference radius with respect to a STx as

$$r_{sp}(d) \equiv \inf\left\{r > 0 | \frac{\rho_p d^{-\alpha}}{\eta + \rho_s r^{-\alpha}} > \beta_p\right\} = \rho_s^{\frac{1}{\alpha}}\left(\frac{\rho_p}{d^\alpha \beta_p} - \eta\right)^{-\frac{1}{\alpha}}.$$
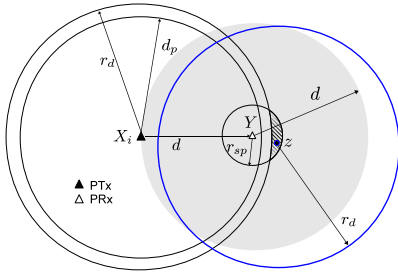
[1]Π_s is independent from Π_p.

Fig. 1: Given no PTx in $b(Y, d)$, the PRx Y can be interfered by potential STxs in hatched region. The activity of potential STx at z is determined by the existence of other PTxs in its detection region $b(z, r_d)$.

Note that $r_{sp}(d)$ is a function of d. As a PRx gets closer to its nearest PTx, r_{sp} gets smaller and the PRx becomes increasingly robust to interference. However, a PRx near the coverage edge is more vulnerable to interference. Third, for a SRx to decode a STx signal, the received SINR at the SRx should be larger than the decoding threshold $\beta_s : \frac{\rho_s d_s^{-\alpha}}{I + \eta} > \beta_s$, from which we define SRx's maximum tolerable amount of interference

$$I_s \equiv \sup \left\{ I > 0 | \frac{\rho_s d_s^{-\alpha}}{I + \eta} > \beta_s \right\} = \frac{\rho_s d_s^{-\alpha}}{\beta_s} - \eta. \quad (1)$$

Fourth, for a SRx to decode a STx signal, the amount of interference from its nearest PTx should be less than the maximum tolerable interference: $\rho_p r^{-\alpha} < I_s$. This leads us to determine a SRx's interference radius with respect to a PTx as

$$r_{ps} \equiv \inf \left\{ r > 0 | \rho_p r^{-\alpha} < I_s \right\} = \left(\frac{\rho_p}{I_s} \right)^{\frac{1}{\alpha}}. \quad (2)$$

Finally, for a SRx to decode a STx signal, the amount of interference from the nearest interfering STx should be less than the tolerable interference: $\rho_s r^{-\alpha} < I_s$. Thus, SRx's interfering radius with respect to a STx is given as

$$r_{ss} \equiv \inf \left\{ r > 0 | \rho_s r^{-\alpha} < I_s \right\} = \left(\frac{\rho_s}{I_s} \right)^{\frac{1}{\alpha}}. \quad (3)$$

Note that I_s, r_{ps}, and r_{ss} above have been selected conservatively.

F. Parameter Sets for Scenarios

Here, we consider following parameter values: $\alpha = 3$, $N_o = -174$dBm, $\eta = N_o \times 20 \times 10^6$, $\rho_s = 1$mW, $\beta_s = 20$, $I_s = 5 \times 10^{-8}$ and $d_s = 10$m, $\rho_p = 100$W, $\beta_p = 10$, $I_p = 5\eta$, $d_p = 27560$m, $r_{ps} = 1259$m, and $r_{ss} = 27$m.

III. PERFORMANCE OF PRIMARY NETWORK

A. Outage Probability of Primary Receiver

In this section, we consider two outage probabilities for a PRx Y; first, the conditional outage probability when the PRx Y is a distance d away from its nearest PTx, which shows how the outage probability changes as d increases; second, the covering probability of primary network. Let $P_{o,1}(d)$ denote the outage probability of a PRx a distance d away from its nearest PTx.

Theorem 1. *(Conditional Outage Probability of a PRx at d from its nearest PTx) For given λ_p, λ_s and d_p, a PRx Y's outage probability given d away from its nearest PTx X_i is*

$$P_{o,1}(d) = 1 - \mathbf{1}_{\{d < d_p\}} \mathcal{L}_{L(d, \Pi_p^{(2)})}(\lambda_s)$$

where $L(d, \Pi) = \int_{b(Y, r_{ps}) \backslash b(X_i, r_d)} \mathbf{1}_{\{z \notin B(\Pi, r_d)\}} dz$, and $\Pi_p^{(2)} = \{\Pi_p \cap \overline{b(Y, d)}\} \cup \{X_i\}$.

See [16] for proof. $\mathcal{L}_{L(d, \Pi_p^{(2)})}(\lambda_s)$ is expected void probability of random subset of gray region in Fig.1 of which area is $L(d, \Pi_p^{(2)})$. Geometrically, the random variable $L(d, \Pi_p^{(2)})$ above denotes a random subset of the set $b(Y, r_{ps}) \backslash b(X_i, r_d)$ which is not covered by the Boolean process $B(\Pi_p^{(2)}, r_d)$. The Laplace transform of $L(d, \Pi_p^{(2)})$ is not easily computable, see [16] for upper and lower bounds. We define the covering probability given as follows.

Definition 1. *(Covering Probability) Define $P_{c,1}(\lambda_p, \lambda_s) \equiv 1 - \mathbb{E}[P_{o,1}(D)] = \int_0^{d_p} P_{o,1}(x) dF_D(x) + \exp\{-\lambda_p \pi d_p^2\}$.*

This covering probability is a metric showing the fraction of area covered by PTxs for given λ_p. So, the higher it is for fixed λ_p, the more efficiently the spectrum is used. Note that the increase of the number of interferers can decrease the covering probability or coverage. So, it will be used later to define the capacity of the primary network in Section V.

IV. PERFORMANCE OF SECONDARY NETWORK

A. Outage Probability of a Typical Secondary Receiver

In this section, we consider the outage probability $P_{o,2}$ of a typical SRx denoted here by W. This is a conditional probability conditioned on the existence of an active STx Z_i transmitting to the SRx W as shown in Fig. 2. Note that Z_i is not necessarily the nearest STx to W. This is the worst case outage probability since we fix $\|W - Z_i\|$ to d_s. For the STx Z_i to be active, there should be no PTxs within STx's detection area; so, we condition on the event $\Pi_p \cap b(Z_i, r_d) = \emptyset$, and $\|W - Z_i\| = d_s$. Note that STx Z_i's detecting the absence of PTxs does not guarantee the successful reception at the SRx W since STx Z_i's detection area may or may not be the super set of SRx W's interference region $b(W, r_{ps})$. So, a potentially harmful PTx can be located there. The interference from other STxs to the SRx W can also cause an outage at the SRx W. The following results captures the impact of the both PTxs and STxs, on the outage of a typical SRx W.

Theorem 2. *(SRx's Conditional Outage Probability) For given λ_p and λ_s, the outage probability of a SRx W a distance d_s away from its STx Z_i is given by*

$$P_{o,2}(\lambda_p, \lambda_s) = 1 - e^{-\lambda_p |b(W, r_{ps}) \backslash b(Z_i, r_d)|} \mathcal{L}_{Q(r_{ss}, \Pi_p^{(3)})}(\lambda_s),$$

where $Q(r, \Pi) \equiv \int_{b(O, r)} \mathbf{1}_{\{z \notin B(\Pi, r_d)\}} dz$, and $\Pi_p^{(3)} = \Pi_p \cap \overline{b(Z_i, r_d) \cup b(W, r_{ps})}$.

V. CAPACITY AND CAPACITY REGION

In this section, we characterize the joint capacity of the primary and secondary networks. The capacity region is of interest since it characterizes all the possible operating regimes. Specifically it is of interest to understand how much capacity
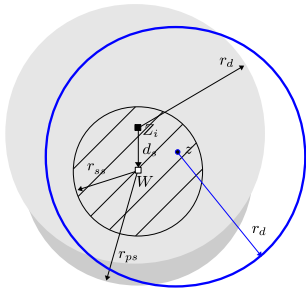
Fig. 2: Conditioned on that there is no PTxs in $b\left(Z_i, r_d\right) \cup b\left(W, r_{ps}\right)$, the outage of a SRx W can be caused by potential STxs in the hatched region $b\left(W, r_{ss}\right)$. Whether a potential STxs, e.g., at location $z \in b\left(W, r_{ss}\right)$, can give harmful interference to W depends on the potential PTxs in $b\left(z, r_d\right) \backslash\left(b\left(Z_i, r_d\right) \cup b\left(W, r_{ps}\right)\right)$.

the secondary network can achieve for a given primary network capacity.

A. Outage Requirement for Secondary Network (ϵ-constraint)

To this end, we first impose an outage constraint on secondary network transmission, called the ϵ-constraint. To support a certain level of QoS, we require the outage be kept low. We will of course find that the capacity region changes as a function of the outage constraint ϵ. We first update the result on the contention density taking into account the ϵ-constraint.

Fact 1. *(Maximum Contention Density under ϵ-constraint) Under an outage constraint ϵ for $P_{o,2}\left(\lambda_p, \lambda_s\right)$, the lower bound of the contention density λ_s^{ϵ} is given as follows by letting $\epsilon = P_{o,2}^u\left(\lambda_p, \lambda_s^{\epsilon,l}\right)$ with $\bar{\epsilon} = 1 - \epsilon$, $k_2 = |b(W, r_{ps}) \backslash b(Z_i, r_d)|$*

$$\lambda_s^{\epsilon,l} = \left[-\frac{k_2}{q}\lambda_p + \frac{1}{q}\log\frac{1}{1-\epsilon}\right]^+ \quad (4)$$

where $q = \mathbb{E}\left[Q\left(r_{ss}, \Pi_p^{(3)}\right)\right] = \int_{b(W, r_{ss})} \exp\left\{-\lambda_p h\left(Z_i, r_d, W, r_{ps}, z, r_d\right)\right\} dz$ *which can be computed numerically. See [16] for details.*

B. Capacity of Primary and Secondary Network

The capacity of the primary network is related to the fraction of covered area (through the covering probability in Definition 1) and the amount of information broadcasted from these stations, which is defined as follows. For given λ_p and λ_s^{ϵ}. The capacity of the primary network C_1 is defined as $C_1\left(\lambda_p, \lambda_s^{\epsilon}\right) = b_p P_{c,1}\left(\lambda_p, \lambda_s^{\epsilon}\right)$. In a similar manner, we can define the capacity for secondary network. It can be understood as the achievable throughput given an outage constraint ϵ, which is defined as $C_{2,\epsilon}\left(\lambda_p, \lambda_s^{\epsilon}\right) = b_s \lambda_s^{\epsilon} P_{tx}\left(1 - \epsilon\right)$, where $P_{tx} = \exp\left\{-\lambda_p \pi r_d^2\right\}$ is the transmission probability of a typical STx. Note that if $|b(W, r_{ps}) \backslash b(Z_i, r_d)| = 0$ for a fixed C_1, the secondary network behaves like a stand-alone ad-hoc network in the sense that the $C_{2,\epsilon}$ increases as ϵ increases until it is maximized at $\epsilon = 1 - \frac{1}{e}$ and will start to decrease as ϵ increases over $1 - \frac{1}{e}$.

C. Capacity Region

Based on the definitions of C_1 and C_2, we now define the capacity region $\mathbb{C}_\epsilon$, which is the set of achievable operating points (C_1, C_2) subject to outage constraint.
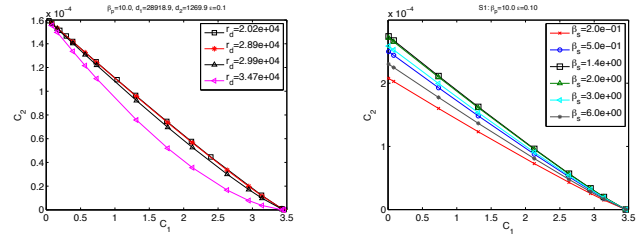
(a) Capacity region was shown for various r_d values. Increasing r_d over $d_1 \approx 2.02 \times 10^4$ decreases $C_2^{\epsilon,l}$ since too large r_d decreases transmission opportunity of STxs.

(b) Capacity region was shown for various β_s values. $C_2^{\epsilon,l}$ is maximized at $\beta_s = \beta_s^*(0)$ for broad ranges of λ_p. The limiting capacity of C_1 corresponds to $\log_2(1 + \beta_p) = 3.46$. Note that as λ_p increases, C_1 increases and $C_2^{\epsilon,l}$ decreases.

Definition 2. *The joint capacity region is defined as*

$$\mathbb{C}_\epsilon = \left\{(x,y) \in \mathbf{R}_+^2 | \exists \lambda_p \geq 0 \text{ s.t. } x = C_1\left(\lambda_p\right), y \leq C_{2,\epsilon}\left(\lambda_p\right)\right\}.$$

VI. IMPACT OF SYSTEM PARAMETERS

A. Impact of Detection Radius and Optimization

We consider the case where we need to determine r_d. Let $d_1 = d_p + r_{sp}(d_p)$ and $d_2 = d_s + r_{ps}$ and suppose that the target decoding SINR of the two networks are given as β_p and β_s. Then, $C_2^{\epsilon,l}$ below is a function of r_d:

$$C_2^{\epsilon,l}(r_d) = \log\left(1 + \beta_s\right) \lambda_s^{\epsilon,l}(r_d)\left(1 - \epsilon\right) \exp\left\{-\lambda_p \pi r_d^2\right\}.$$

Recall that $\lambda_s^{\epsilon,l}$ in (4) has $k_2 \equiv |b(W, r_{ps}) \backslash b(Z_i, r_d)|$ term, which is a function of r_d. Suppose $r_d < d_2$. Then, increasing r_d makes $k_2 \to 0$, which consequently reduces harmful interference from hidden PTxs and accordingly the outage probability decreases. Thus, increasing $r_d(< d_2)$ increases $\lambda_s^{\epsilon,l}$. Note that q is also a decreasing function of r_d but hardly changes. Once if $r_d \geq d_2$, then, we have $k_2 = 0$ and $\lambda_s^{\epsilon,l}$ increases very slowly and looks constant. We observe that if $r_d < d_2$ increasing $\lambda_s^{\epsilon,l}$ dominates decreasing $\exp\{-\lambda_p \pi r_d^2\}$, which makes $C_2^{\epsilon,l}$ increasing. While if $r_d \geq d_2$ the latter dominates and $C_2^{\epsilon,l}$ starts to decrease. So, from the perspective of reducing the impact from hidden PTxs, $r_d = d_s + r_{ps}$ is a near optimal choice so as to maximize capacity. But along with maximizing $C_2^{\epsilon,l}$ it is also necessary to protect primary receivers (note that this maximizes C_1). So, r_d should be chosen as follows.

Rule of Thumb 1. *(RT1) For given $d_1 = d_p + r_{sp}(d_p)$ and $d_2 = d_s + r_{ps}$, choose the the detection radius of STxs as $r_d = \max\{d_1, d_2\}$. It is the sub-optimal choice for maximizing secondary capacity.*

Increasing r_d further is not helpful to increasing capacity since it exponentially reduces transmission opportunities.

Fig.3a shows the change in the capacity region for various values of r_d when $d_1 = 2.02 \times 10^4$ and $d_2 = 1269.9$. Since $d_2 < d_1$ in our setting, increasing r_d decreases $C_2^{\epsilon,l}$ due to decreasing transmission opportunity.

B. Impact of Decoding SINR and Optimization

Consider the case where we need to determine β_s given all other parameters except r_d which is a function of β_s by RT1. The transmission capacity of secondary network depends on

several terms which increase ($\nearrow$) and decrease ($\searrow$) with β_s as follows:

$$C_2^{\epsilon,l}(\underbrace{\beta_s}_{}) = c_2 \underbrace{\log(1+\beta_s)}_{\nearrow} \underbrace{(1/q(\beta_s))}_{\nearrow} \underbrace{\exp\{-\lambda_p \pi r_d(\beta_s)^2\}}_{\searrow}$$

(5)

for some constant $c_2 > 0$. Note that as β_s increases $r_{ss} \approx d_s \beta_s^{1/\alpha}$ increases, and accordingly $q(\beta_s)$ also increases. Also as β_s increases, r_{ps} increases, which eventually increases r_d since r_d is assumed to be selected according to the RT1. This exhibits the tradeoff between the three terms in (5). *Increasing the transmission rate (or increasing β_s) makes the SRx more sensitive to interference and accordingly under a fixed ϵ allows fewer concurrent transmitters (decreasing node density) and may discourage transmission attempts(decreasing transmission opportunity).* Since $C_2^{\epsilon,l}(\beta_s)$ is a product of both increasing and decreasing terms there exists a unique maximum point β_s^* maximizing $C_2^{\epsilon,l}$, which is a function of λ_p. For $\lambda_p = 0$, we can analytically find β_s^* after ignoring noise term. Setting $dC_2^{\epsilon,l}/d\beta_s = 0$ gives $\beta_s^*(0) = \exp\left\{W_0\left(-\frac{\alpha}{2e^{\alpha/2}}\right) + \frac{\alpha}{2}\right\} - 1$ for $\alpha > 2$. For $\lambda_p > 0$, $\beta_s^*(\lambda_p)$ can be found numerically.

Suppose that a minimum required data rate (or equivalently decoding SINR) for secondary node's applications is specified as a design requirement, denote it by β_s^m. Then, from the above discussion, it follows that there exists an optimal decoding SINR $\beta_s^*(\lambda_p)$, which suggests following.

Rule of Thumb 2. *(RT2) For a given λ_p and an application-required decoding SINR β_s^m, the operating decoding SINR chosen as $\beta_s^o = \max\{\beta_s^*(\lambda_p), \beta_s^m\}$ maximizes the secondary capacity while satisfying the application requirement.*

Replacing $\beta_s^*(\lambda_p)$ in RT2 with $\beta_s^*(0)$ makes the rule of thumb easy to use but gives sub-optimal performance. Fig. 3b shows the changes in the capacity region under various β_s. By the definition of $\beta_s^*(\lambda_p)$, $C_2^{\epsilon,l}$ is maximized at $\beta_s = \beta_s^*(\lambda_p)$ for given λ_p. However, we have $\beta_s^*(\lambda_p) \approx \beta_s^*(0) = 1.4$ for broad ranges of λ_p in Fig.3b. Note that the capacity region is roughly bounded by linear boundary, this is because we have $d_p \approx r_d$, then $P_{c,1} \approx e^{-\lambda_p \pi r_d^2}$ and from the definition of C_1 and $C_2^{\epsilon,l}$, it is straightforward to show the linear relationship.

C. Impact of Transmit Power of STxs and Optimization

In this section, we show the existence of an optimal transmit power for secondary nodes which maximizes the secondary capacity. An approximation of the optimal transmit power is provided.

We make following assumptions.

(A1) *Let the detection radius of STxs be determined as $r_d = \max\{d_p + \kappa r_{sp}(d_p), d_s + r_{ps}\}$ for some $\kappa \geq 0$.*[2]
(A2) *Assume that ϵ-contention density λ_s^ϵ is a constant with respect to d_p and ρ_s, though it changes slowly as a function of them.*
(A3) *Assume that it is required by system design requirements that secondary nodes' tolerable interference level should be at least $I_s^{\min}$, which consequently determines the minimum required transmit power $\rho_s^{\min} = \inf\left\{\rho_s > 0 \mid \frac{\rho_s d_s^{-\alpha}}{\eta + I_s^{\min}} > \beta_s\right\} =$*

$\beta_s d_s^\alpha (\eta + I_s^{\min})$.[3]

λ_s^ϵ is a constant due to (A2) and thus we optimize $P_{tx}(\rho_s)$ over ρ_s to maximize $C_2^{\epsilon,l}$. Note that $r_d = \max\{d_1, d_2\}$ chosen by (A1) is a function of ρ_s. Specifically, $d_1(\rho_s) = d_p + r_{sp}(d_p, \rho_s)$ is a monotonically increasing function of ρ_s, while $d_2(\rho_s) = d_s + r_{ps}(\rho_s)$ is a monotonically decreasing function of ρ_s. Thus, there exist an optimal ρ_s minimizing $r_d(\rho_s)$. Note that minimizing detection radius r_d maximizes the transmission probability P_{tx}, and accordingly maximizes $C_2^{\epsilon,l}$. Let ρ_s^* be the optimal transmit power, then $d_p + r_{sp}(d_p, \rho_s^*) = d_s + r_{ps}(\rho_s^*)$ holds. Since it is hard to find a closed form expression for ρ_s^*, we find an approximation $\hat{\rho}_s^*$ using the fact that $d_p + r_{sp}(d_p, \rho_s^*) \approx d_p$. With the approximation, solving $r_{ps}(\rho_s^*) \approx d_p - d_s$ gives $\rho_s^* \approx \beta_s d_s^\alpha \left(\eta + \frac{\rho_p}{(d_p - d_s)^\alpha}\right)$. Then considering the minimum required transmit power, we have an approximated value of ρ_s^* given as follows.

Rule of Thumb 3. *(RT3) For a given secondary system design requirements β_s, d_s and $\rho_s^{\min}$, choose the transmit power of secondary node as $\hat{\rho}_s^* = \max\left\{\rho_s^{\min}, \beta_s d_s^\alpha \left(\eta + \frac{\rho_p}{(d_p - d_s)^\alpha}\right)\right\}$. It is a sub-optimal choice for maximizing the secondary capacity.*

Fig.4a shows $C_2^{\epsilon,l}$ as a function of ρ_s, which is maximized at $\rho_s = \rho_s^*$. The vertical line denotes the approximation $\hat{\rho}_s^*$, which is quite close to the optimal value. If $\rho_s^* < \rho_s$, increasing transmit power ρ_s increases detection radius $r_d = d_p + r_{sp}(d_p, \rho_s)$ and makes it more conservative, which accordingly results in a capacity loss. While if $\rho_s < \rho_s^*$, decreasing transmit power ρ_s increases detection radius $r_d = d_s + r_{ps}(\rho_s)$ since decreasing transmit power ρ_s makes SRx more vulnerable to the interference from PTxs, similarly which causes the loss of secondary capacity.

D. Impact of Coverage's Burstiness on Secondary Capacity

In this section, we show how the burstiness of a primary network's coverage affects the capacity of an associated secondary network For that end, we define the notion of burstiness for Boolean process and make assumptions for simple analysis.

We adopt the definition of burstiness introduced in [17]. For two given primary networks A and B with the same fixed coverage c, we say that the Network A has a more bursty coverage than the Network B if the Network A has a larger coverage radius than that of the Network B. Fig. 3 shows the realizations of two primary networks' coverage with the same coverage area, where the union of bright gray discs is the coverage of PTxs and the union of dark gray regions around it is the guard band to protect PRxs from STxs. The thickness of the band is given as $\kappa r_{sp}(d_p)$ for given $r_{sp}(d_p)$ and $\kappa \geq 0$ is a measure of the conservativeness of detection radius. If $\kappa = 0$, there is no guard band, otherwise the guard band is chosen conservatively.

In this section, we need assumptions (A1) and (A2) with following additional assumption.

(A4) *Assume the primary networks of interest have the fixed*

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[2]In Section VI-C, we assume $\kappa = 1$.

[3]Note that ρ_s, d_s, and I_s have dependency among them. In previous sections, $I_s^{\min}$ was the function of given ρ_s and d_s. While in this section $I_s^{\min}$ and d_s are given, which accordingly determines $\rho_s^{\min}$.

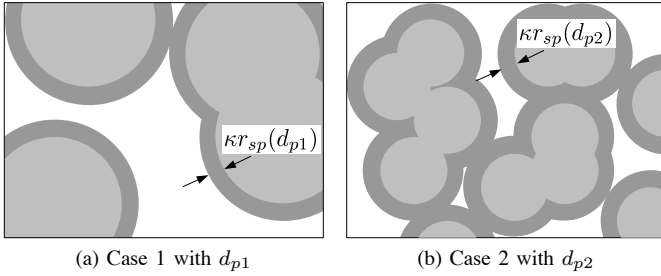(a) Case 1 with d_{p1} (b) Case 2 with d_{p2}

Fig. 3: Two realizations of primary network with the same coverage but different coverage radii were shown. Coverage and guard region were shown as bright and dark gray region respectively. Note that the thickness of guard band is the same, i.e., $\kappa r_{sp}(d_{p1}) = \kappa r_{sp}(d_{p1})$.

(a) There exist an optimal ρ_s^* which maximizes $C_2^{\epsilon,l}$. ρ_s^* can be approximated by $\hat{\rho}_s^*$ maximizing transmission probability of STxs. Vertical line denotes $\hat{\rho}_s^*$.

(b) For a fixed coverage ($c = 0.5$), a primary network with more bursty coverage and less conservative detection radius allows higher secondary capacity C_2.

Fig. 4

coverage fraction $0 < c < 1$, i.e., $P(O \in B(\Pi_p, d_p)) = c$ which gives a following condition: $1 - \exp\{-\lambda_p \pi d_p^2\} = c$.

We need the assumption (A2) to make the optimization process simple. Note that $C_2^{\epsilon,l}$ is proportional to both λ_s^ϵ and P_{tx}, where both terms are the functions of d_p. However, we maximize P_{tx} only over d_p since λ_s^ϵ varies slowly over d_p. With the above settings, we have following observations.

Proposition 1. *Under the assumptions, if $r_d = d_p$ (or $\kappa = 0$), then the capacity of the secondary network is not affected by the burstiness of the primary network's coverage (or d_p). If $r_d > d_p$ (or $\kappa > 0$), then the capacity of the secondary network decreases as the primary network's coverage gets less bursty. The capacity decrease depends on the conservativeness of detection radius κ.*

Note that $\kappa = 0$ implies there is no conservativeness in detection radius and no guard bands, then it is straightforward to see that $C_2^{\epsilon,l}$ is a constant since λ_s^ϵ and P_{tx} are constants by (A2) and (A4) respectively. However, if $\kappa > 0$, the conservativeness of the detection radius affects $C_2^{\epsilon,l}$. Intuitively, this happens because the area consumed by the guard band increases as the primary network becomes less bursty (or smaller d_p), which results in a smaller transmission probability. That is, $P_{tx} = \exp\{-\lambda_p \pi r_d^2\} = (1 - c)^{\gamma^2}$ with $\gamma = 1 + \frac{\kappa r_{sp}(d_p)}{d_p}$ decreases as d_p decreases. If d_p approaches to 0, then all the non-covered region is used for guard band purpose and there is no room for secondary nodes to operate, leading to zero capacity ($C_2^{\epsilon,l} = 0$). Fig. 4b shows the relation between burstiness and capacity under various conservativeness. It is clearly shown that in general more bursty network has higher $C_2^{\epsilon,l}$ and less conservative detection radius admits higher capacity $C_2^{\epsilon,l}$. Note that $\kappa = 0$ case has almost flat capacity $C_2^{\epsilon,l}$, which makes the assumption (A2) valid.

VII. CONCLUDING REMARKS

We have explored the interdependency between the primary and secondary networks with different access priorities to single frequency band in terms of the outage probability and joint capacity region. The model suggests that the detection radius(or detection sensitivity) of cognitive device needs to be determined carefully not only to protect primary receivers but also to minimize the impact from hidden primary transmitters. Along with this, we have shown that there exists an optimal decoding SINR and transmit power of cognitive devices that maximizes the capacity of cognitive network. Furthermore we
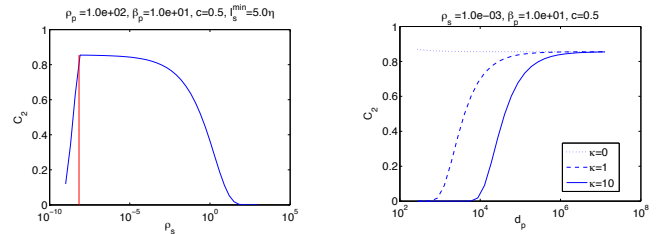
show that primary networks with bursty coverage admit higher secondary capacity. We note that these parameters (except primary transmit power) are easily adjustable without requiring complex algorithms or hardware modification.

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